APPENDIX A
TECHNICAL APPENDIX

Properties of Indifference Curves

Except in special cases, indifference curves have the following properties:[1]

- *They are negatively sloping*. This reflects our basic assumption that the worker places a positive value on both types of job characteristics. (This is the fundamental characteristic of a *good*.) Hence, if the worker gets more of one good, he or she must give up some amount of the other in order to remain on the same indifference curve, and vice versa.
- *They do not touch or intersect another indifference curve.* If they did, it would imply a logical contradiction in the worker's behavior. For example, consider the two indifference curves shown in Figure A-1. These curves imply that the worker is indifferent between combinations *a* and *b*, and between *a* and *x*. Hence, transitivity of choices requires that he or she be indifferent between *x* and *b*. However, observe that *x* contains more of both job characteristics than *b* does. Therefore, the worker must prefer *x* to *b*, contradicting our previous conclusion that he or she is indifferent between them. Because it is logically impossible for the worker to simultaneously prefer *x* to *b* and be indifferent between them, we must rule out the possibility that indifference curves may touch or intersect each other.

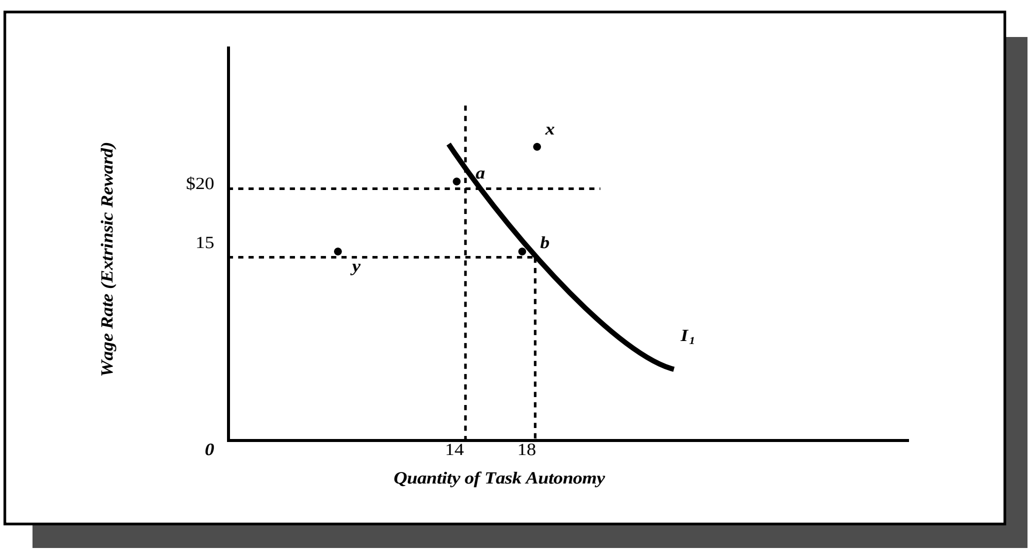

Figure A-1. Intersecting Indifference Curves

[1] Example special cases include negatively sloped, straight-line indifference curves (namely, the two goods are perfect substitutes), L-shaped curves (perfect complements), and perfectly horizontal or vertical indifference curves (one good or the other has zero marginal utility).

- *They are convex to the origin.*[2] Think of the worker as moving (voluntarily) from combination *a* to combination *b* along indifference curve I_1 in Figure A-1. As he or she does so, the greater task autonomy he or she receives increases the amount of task autonomy he or she enjoys from this source. Recall that the law of diminishing marginal utility implies that, other things held constant, equal increments of task autonomy will yield smaller and smaller increases in total satisfaction. At the same time, the worker is (voluntarily) accepting a lower wage rate. As the wage rate falls, and with it his or her ability to buy other goods and services, the marginal utility of wages would increase. Combining these two observations implies that, as the worker moves from upper left to lower right along an indifference curve, the amount of wages he or she would voluntarily give up in exchange for an additional unit of task autonomy would decrease. That is, moving from upper left to lower right, the slope of the indifference curve must get smaller and smaller (in absolute value). This requires that the indifference curve be convex to the origin.

The Budget Line

Geometric Interpretation

Denote the quantity of task autonomy at any point by R, the wage rate by W, the constant price of an additional unit of task autonomy by p_R, and the constant price of an additional "unit" of wages by p_W. Assume that the worker's compensation budget is fixed at CB_1. If the individual were to take his or her entire compensation budget in the form of wages, and use none of it to "buy" more job autonomy, he or she would receive a money income equal to CB_1 / p_W. This is the vertical intercept of the budget line, as shown in Figure A-2. Similarly, if the worker were to "spend" his or her entire compensation budget on job autonomy (and, hence, receive no money income), he or she could buy a number of "units" of job autonomy equal to CB_1 / p_R. This is the budget line's horizontal intercept. As demonstrated below, the slope of this isocost line is $-(p_R / p_W)$. By noting that p_W, the "price" of an additional "unit" of wages, is \$1, we can simplify the slope to $-p_R$, which is just the price of an additional unit of task autonomy. Similarly, the vertical intercept becomes just CB, the value of the compensation budget.

For the budget CB_1, the worker could afford any combination of wage rates and job autonomy along the straight line connecting these two end-points, as shown in Figure A-2. Note that the higher the compensation budget CB, the farther from the origin the budget line will be.

[2] This economic property is known as the law of diminishing marginal rate of substitution (MRS). At any point on an indifference curve, the MRS is defined as the absolute value of the slope of the indifference curve at that point. A full explanation of this property requires some differential calculus and is beyond the scope of this paper. For a more advanced treatment, see Miltiades Chacholiades, *Microeconomics* (New York, NY: Macmillan, 1986), pp. 93-96, 555-557; and Alpha C. Chiang, *Fundamental Methods of Mathematical Economics*, 3d ed. (New York, NY: McGraw-Hill, 1984), pp. 400–404.

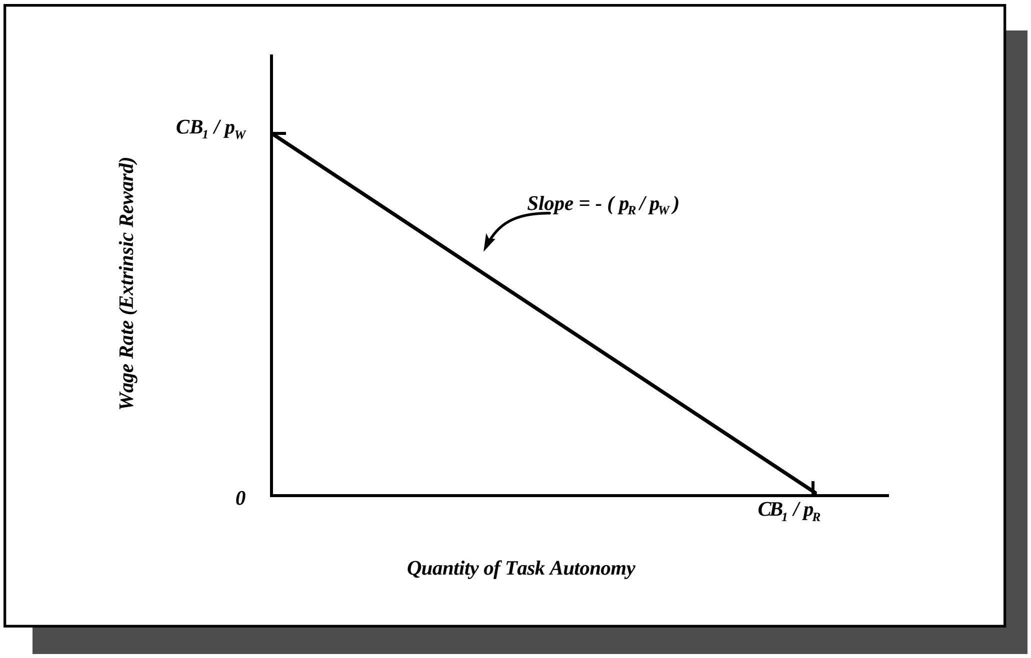

Figure A-2. A Compensation Budget Constraint

Algebraic Derivation

In general, the compensation budget *CB* required to buy any combination of task autonomy and wages is given by

$$CB = p_R \bullet R + p_W \bullet W.$$

Solving for W gives the equation of any straight line passing through any point, like point *a* in Figure 6-4:

$$W = -(p_R / p_W) \bullet R + (CB / p_W).$$

The slope of this line is $-(p_R / p_W)$, the coefficient of the *R* term, and the vertical intercept is (CB / p_W). (Its horizontal intercept is (CB / p_R), as shown in Figure A-2.) Again, by noting that the "price" of an additional "unit" of wages is \$1, we can simplify the slope to $-p_R$, which is just the price of an additional unit of task autonomy. Similarly, the vertical intercept becomes just *CB*, the value of the compensation budget at any point along this straight line.

Note that from the organization's perspective, the value of the worker's compensation budget is the organization's total cost of providing this combination of wages and task autonomy. This demonstrates that any combination of these two job characteristics lying on a straight line would have the same total cost *CB*, given p_R, as asserted in the discussion of Figure 6-4.[3]

Nonlinear Budget Constraints

To simplify the illustration above, we have assumed that the price of an additional "unit" of task autonomy is constant. This might not be the case. In some instances, the process of producing the job characteristics (like task autonomy) that provide the basis for intrinsic rewards may be subject to the law of diminishing marginal productivity. For example, in redesigning jobs to provide greater potential for intrinsic rewards for workers, an organization that is acting rationally would first make those changes that are likely to have the greatest effect per dollar spent on them. Hence, subsequent changes would probably yield fewer benefits per dollar expended on them than the initial redesigns did. As a result, the marginal cost of producing additional units of nonwage job characteristics would increase. Given the fixed compensation budget associated with any compensation budget line, this means that wages would have to fall by larger and larger amounts to offset the higher cost of producing additional units of nonwage job characteristics. Hence, the marginal cost of an additional unit of such job characteristics would increase, and the budget line would be concave to the origin, as shown in Figure A-3.

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[3] Hence, from the organization's perspective, this line would be called an isocost, or equal-cost, line. Economics texts normally use the term isocost line in the context of an organization's resource-allocation decisions, for example, buying combinations of inputs for its production process. In the context of a person's decisions about choosing among alternative combinations of goods and services, or among alternative combinations of compensation elements, the analogous concept is the budget line. Algebraically, the two are identical; all the properties discussed above apply to both. Nevertheless, there is a significant conceptual distinction between the two. Given his income from all sources, we can treat a person's total budget for any given period of time as fixed. (It is fairly easy to adapt the model to include apparent exceptions to this general conclusion. "Income from all sources" includes monetary gifts. We can also consider it to include any borrowing or dissaving, which would be limited by the person's credit limit, accumulated wealth, and his other financial characteristics.) We can think of the person's objective as attempting to maximize his satisfaction (namely, to reach the highest indifference curve possible), subject to his income or budget constraint.

On the other hand, if an organization is deciding which level of output to produce (thus, which combination of inputs to hire to produce that output), its total cost budget is one of the results of the decision process, not a given parameter. That is, the firm must incur the cost necessary to buy the combination of inputs required to produce its desired level of output, and the cost will vary with the level of output. If the organization wants to increase its output level, it must hire more inputs and incur the correspondingly higher costs, other things being equal. Using budget line to refer to individuals and isocost for organizations recognizes this practical distinction, even though the two are mathematically equivalent.

The discussion in the preceding paragraph gives the constrained minimization form of the production problem: given the desired level of output to be produced, the organization should choose the combination of inputs that minimizes the total cost of producing that level of output. The mathematical dual of this form is the constrained maximization problem: given a total cost budget, the organization seeks the combination of inputs that will maximize the quantity of output it can produce with that budget. Mathematically, the two problems are equivalent to each other.

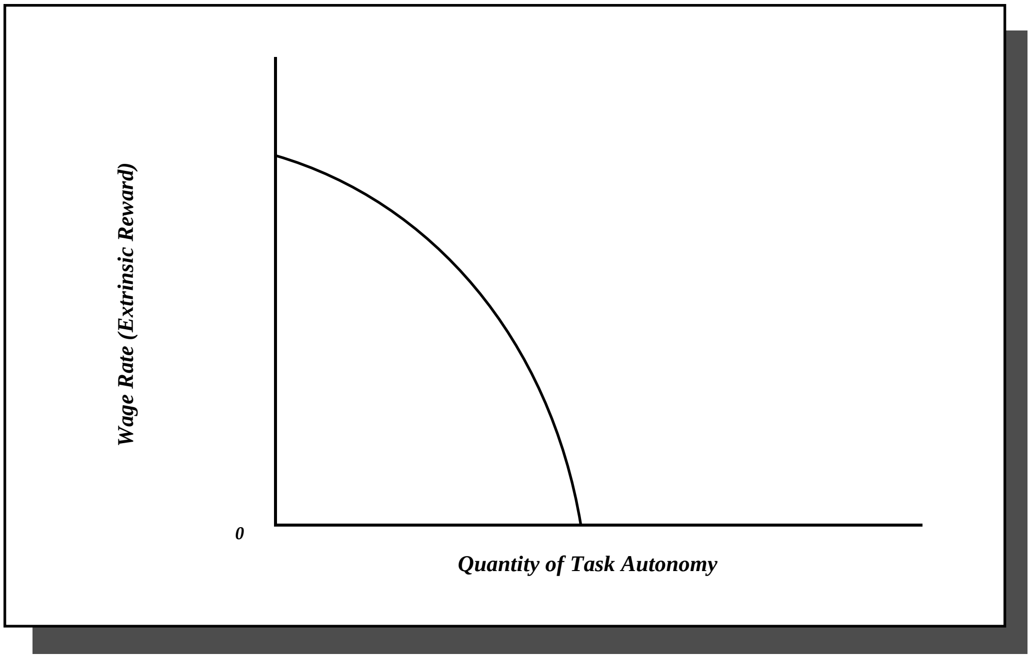

Figure A-3. Nonlinear Compensation Budget Constraint

The Optimal Compensation Package

Graphical Solution

Recall that a worker's budget line shows the various combinations of job characteristics he or she can afford, given his or her compensation budget, while his or her indifference map shows the amount of satisfaction that any combination of job characteristics gives him or her. We can think of a combination of these job characteristics as being the components of a compensation package. Figure A-4 combines one of the worker's possible budget lines with his or her indifference map to demonstrate his or her *optimal compensation package*, namely, the combination of compensation elements that gives him or her the *maximum* satisfaction possible from his or her compensation budget.

To maximize his or her satisfaction, the worker must choose the combination of compensation elements that puts him or her on the highest indifference curve he or she can reach, given the size of his or her compensation budget. Graphically, his or her optimal combination is shown by point *a*, where the budget line is tangent to the highest possible indifference curve, I_2 in this case.[4] By selecting the combination represented by wage rate W_1 and task autonomy level R_1, the worker maximizes the satisfaction he or she receives from his or her given compensation budget.

[4] It is purely coincidental that point a lies at approximately the midpoint of the budget line. The location of the tangency point will depend on the slopes of the budget line and indifference curves.

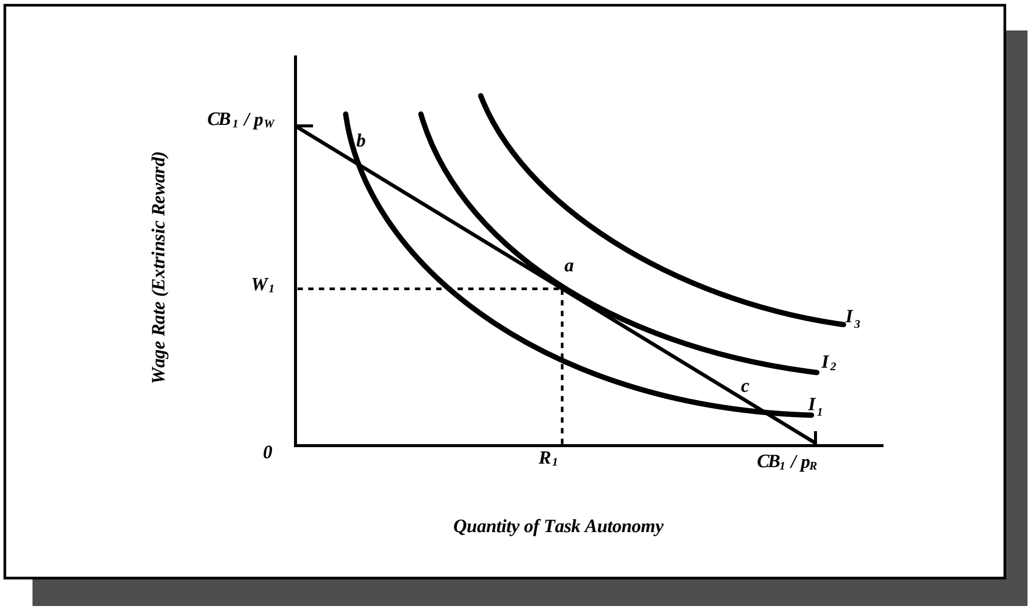

Figure A-4. The Worker's Optimal Compensation Package (Linear Budget Constraint)

Note that the worker could also afford the combinations represented by points *b* and *c*, because they both lie on the same budget line as *a*. However, combination *a* is on a higher indifference curve than any other combination on this budget line. Hence, the worker would prefer combination *a* to all the other combinations he or she could afford, like *b* and *c*. In addition, note that the worker would clearly prefer to be able to reach an even higher indifference curve than I_2, but he or she can't afford to do so, given compensation budget CB_1.[5]

Mathematical Solution

In general, the worker's optimal compensation package is given by the solution to a constrained optimization problem, in which we seek to maximize the worker's satisfaction subject to his or her budget constraint. Let $U = U(W, R)$ describe the worker's utility, or preference, function.[6] We want to maximize this objective function subject to the compensation budget constraint

$$CB = p_W \cdot W + p_R \cdot R,$$

where p_W and p_R are constants.[7]

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[5] Because individual workers are likely to differ in their tastes for wages and nonwage job characteristics, their indifference maps would differ from the one shown in Figure A-4. Hence, their tangency points, representing their respective optimal compensation packages, would likely occur at a different combination of compensation elements than point *a*.

[6] The utility function and the subsequent first- and second-order conditions can be extended to include any number of variables.

[7] As discussed previously, if providing workers with job autonomy and other intrinsically rewarding aspects of the job is subject to the law of diminishing returns, the price the organization "charges" for additional units of such intrinsically rewarding job characteristics isn't likely to be constant over wide ranges. As explained above, this price would increase as the quantity provided increases. In this case, $p_R = f(R)$, with $df(R)/dR > 0$. In the first- and second-order conditions below, the constant p_R term should be replaced with $df(R)/dR$.

The first-order conditions for maximizing $U = U(W, R)$ are given by:

$$\frac{LU / LW}{p_W} = \frac{LU / LR}{p_R}$$

The values of W and R that satisfy the first-order conditions correspond to the tangency point in Figure A-4.

The second-order conditions require that the following bordered Hessian determinant be positive:

$$\begin{matrix} 0 & p_W & p_R \\ p_W & U_{WW} & U_{WR} \\ p_R & U_{RW} & U_{RR} \end{matrix}$$

$$= 2p_W p_R U_{WR} - p_R^{\,2} U_{WW} - p_W^{\,2} U_{RR} > 0,$$

with all the derivatives evaluated at the values of W and R that satisfy the first-order conditions, for example, W_1 and R_1 in Figure A-4. Geometrically, the second-order conditions mean that the indifference curves must be convex to the origin at the optimal combination of compensation elements. Together, the first- and second-order conditions constitute the necessary and sufficient conditions for a local maximum of the utility function, $U = U(W, R)$.

Changes in the Compensation Package

As mentioned in the preceding discussion, increases in the level of the worker's compensation budget shift the budget line outward; decreases shift it inward. Because the slope of the budget line is determined by the price ratio $-(p_R / p_W)$, the new budget lines would be parallel to the original one as long as the relative prices of the two compensation components remain constant. Figure A-5 shows three possible budget lines, CB_1, CB_2, and CB_3, drawn so they are tangent to three of the worker's indifference curves. Given this worker's preference map, note that he or she responds to increases in his or her compensation budget by increasing the amount he or she "consumes" of both types of job characteristics.[8]

[8] It is theoretically possible that, beyond a certain level of income, he might decrease his consumption of one of the job characteristics if his income increases. For a normal good, consumption increases with income, while it decreases for an inferior good. (Example inferior goods include used clothes and cars, hamburger meat, canned processed meat parts, and generic products.)

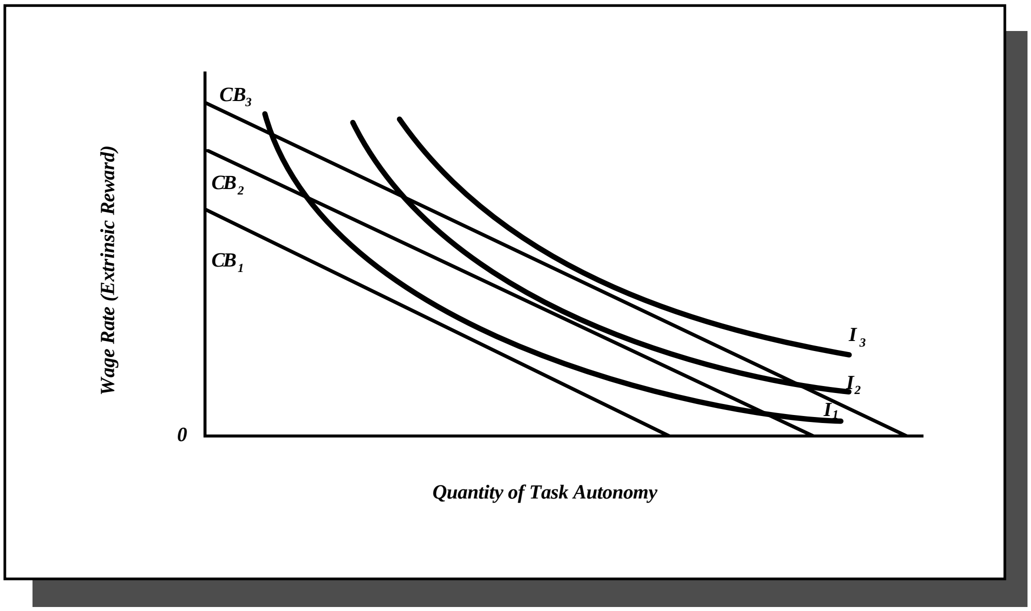

Figure A-5. Isoquant and Budget Constraint Maps (Linear Budget Constraints)

The organization that employs the worker typically determines the compensation budget constraint the worker faces.[9] *If the organization is already operating as efficiently as possible*, increases in the total value of compensation increase the organization's operating costs, other things being equal. From the organization's perspective, the worker's compensation budget line is a compensation *isocost* line. Hence, the organization's operating budget will constrain the level of compensation it can offer its workers. (In a for-profit firm, compensation increases would decrease the firm's profits, other things being equal, and the firm's target profit level would constrain the compensation it offered.)[10]

[9] Even in those team-based pay schemes that allow team members to determine the compensation of each team member, we can think of the decision as being made by team "organization," subject to the compensation budget it is given by the parent organization.

[10] The compensation budget lines can also be viewed as isoprofit, or same-profit, lines. Because higher compensation budget lines represent higher total compensation costs, they also represent lower profit levels for the firm.